DATA X

Data as a Signal and Correlation
Data, Signals, and Systems

Ikhlaq Sidhu
Chief Scientist & Founding Director,
Sutardja Center for
Entrepreneurship & Technology
IEOR Emerging Area Professor Award,
UC Berkeley

Berkeley SCET

Converting From Time Sequence Data to Features

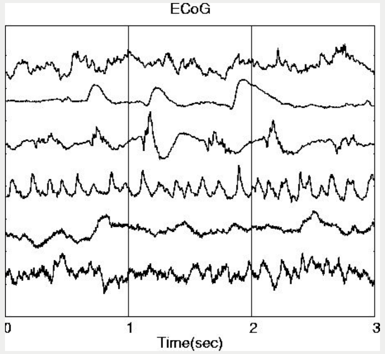
Of course, not all data has a time property, but lets start with this type.
For example(key1, value 1),(key 2, value 2)… in this case, the keys are indexed by time.

Berkeley SCET

Converting From Time Sequence Data to Features

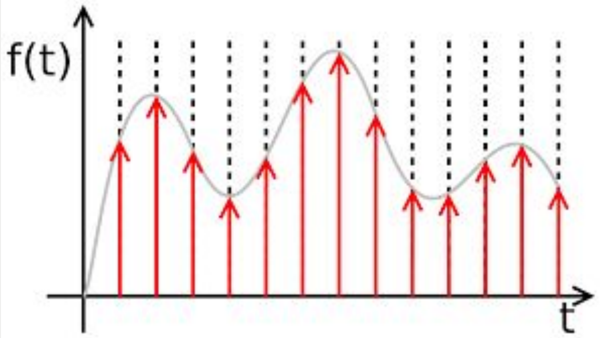
Many Types of data
are signals in time

- Stock market
- Temperature
- Instrument readings



Continuous signals x(t)

Sometimes we sample them,
record at intervals of T



Sampled signals (data) x(nT)

We get a list in a table,
array, or vector

Rec	Observed
1	60.323
2	61.122
3	60.171
4	61.187
5	63.221
6	63.639
7	64.989
8	63.761
9	66.019
10	67.857
11	68.169
12	66.513
13	68.655
14	69.564
15	69.331
16	70.551

Discrete data
x_n = x1, x2, x3, …

(might lose time reference)

What we want (for now):
features and characteristics

For example:

- Means
- Variances
- Patten matches
- Changes
- accumulation
- Frequency

Berkeley SCET

What is the Correlation of the table?

	A	B	C	D
Date	Ozone ($\mu g/m^3$)	Temperature (°C)	Relative humidity (%)	n deaths
1 Jan 2002	4.59	−0.2	75.7	199
2 Jan 2002	4.88	0.1	77.5	231
3 Jan 2002	4.71	0.9	81.3	210
4 Jan 2002	4.14	0.5	85.4	203
5 Jan 2002	2.01	4.3	93.5	224
6 Jan 2002	2.4	7.1	96.4	198
7 Jan 2002	4.08	5.2	93.5	180
8 Jan 2002	3.13	3.5	81.5	188
9 Jan 2002	2.05	3.2	88.3	168
10 Jan 2002	5.19	5.3	85.4	194
11 Jan 2002	3.59	3.0	92.6	223
12 Jan 2002	12.87	4.8	94.2	201

Leads to question:

What does it mean for one row to be similar to another?

Is what is the Correlation (A, B)

Correlation Matrix: Or how is every column related to every other column:

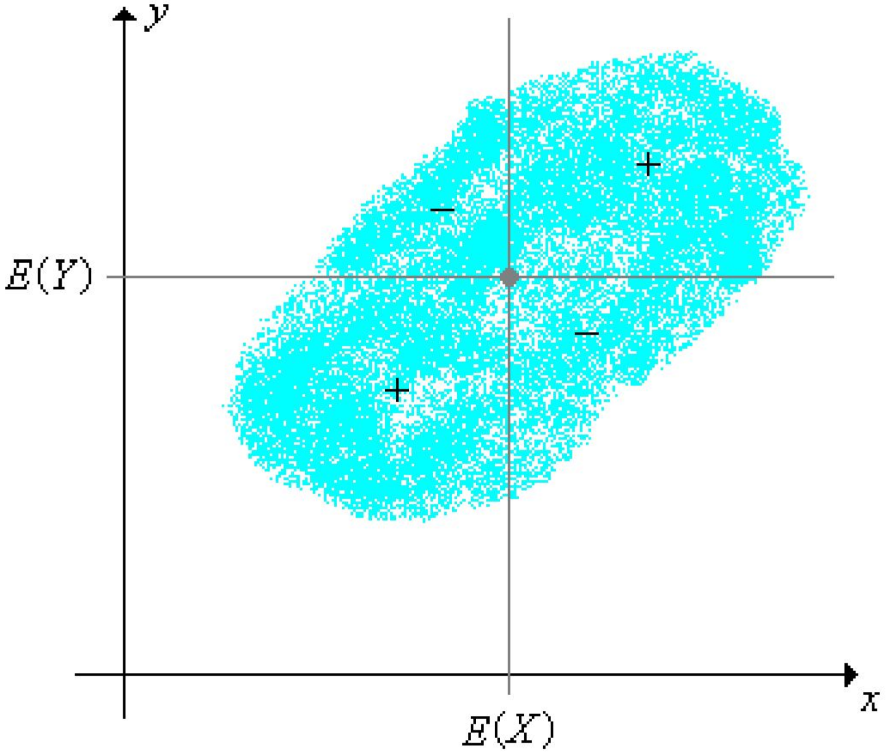
AA	AB	AC	AD
BA	BB	BC	BD
CA	CB	CC	CD
DA	DB	BC	DD

Correlation and Correlation Matrices

Berkeley SCET

Correlation and Covariance:

A practical example



Data Table

X	Y
5	7
8	10
14	7
15	12
....	...
...	...
...	...
...	...
...	...

$$\operatorname{cov}(X,Y) = \mathbb{E}\left([X-\mathbb{E}(X)]\,[Y-\mathbb{E}(Y)]\right)$$

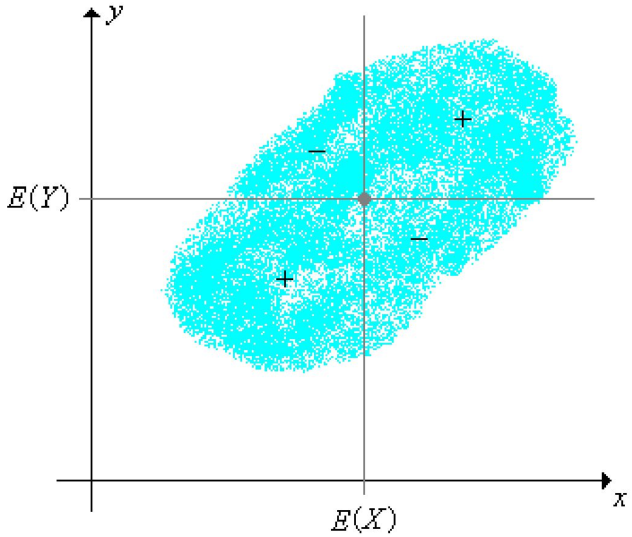
$$\rho_{X,Y} = \operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$= \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y},$$

Berkeley SCET

Correlation and Covariance:

A practical example



Data Table

X	Y
5	7
8	10
14	7
15	12
....	...
...	...
...	...

$$\operatorname{cov}(X,Y) = \mathbb{E}\left([X-\mathbb{E}(X)]\,[Y-\mathbb{E}(Y)]\right)$$

$$\rho_{X,Y} = \operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$= \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y},$$

$$|Cov(X,Y)|^2 \le Var(X)Var(Y)$$

$$\therefore\ |Cov(X,Y)| \le \sqrt{Var(X)Var(Y)}$$

plug this result from the Cauchy-Schwarz

$$|\rho| = \left|\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}\right| \le \frac{\sqrt{Var(X)Var(Y)}}{\sqrt{Var(X)Var(Y)}} = 1$$

Berkeley SCET

Example:

What is the correlation of X,Y?

How Do we Find It?

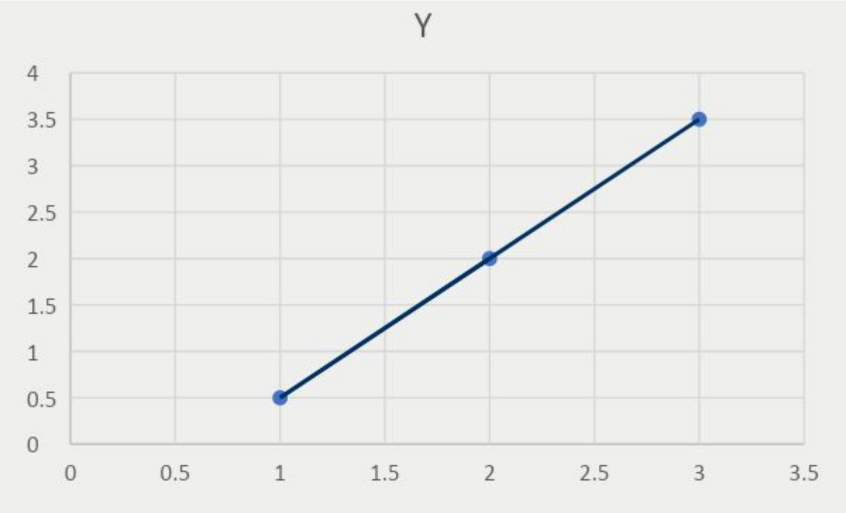
X	Y
1	0.5
2	2
3	3.5

Example:

What is the correlation of X,Y?

How Do we Find It?

X	Y
1	0.5
2	2
3	3.5



Example:

What is the correlation of X,Y?

How Do we Find It?

	X	Y
	1	0.5
	2	2
	3	3.5
Mean	2.00	2.00
Standard Deviation	1	1.5
Variance	1	2.25

Example: What is the correlation of X,Y?

One way to do it:

	Example				
	X	Y		X*Y	E[X]E[Y]
	1	0.5		0.5	4
	2	2		4	4
	3	3.5		10.5	4
mean	2.00	2.00		5.00	4.00
stdev	1	1.5			
var	1	2.25			

$$\text{Corr}(X,Y) = \frac{E[XY] - E[X]E[Y]}{\text{stdev}(X) * \text{stdev}(Y)}$$

$$= E[XY] - E[X]E[Y] / 1.5$$

$$= \quad 5 - 4 / 1.5$$

$$= 1 / 1.5 = .67$$

Example: What is the correlation of X,Y

The other way to do it

	X	Y		X-ux	Y-uy	(X-ux)(Y-uy)
	1	0.5		-1	-1.5	1.5
	2	2		0	0	0
	3	3.5		1	1.5	1.5
mean	2.00	2.00		0.00	0.00	1.00
st.dev	1	1.5				

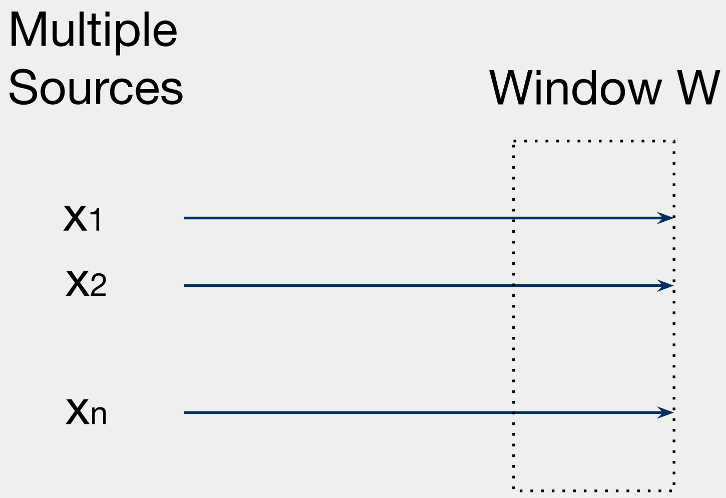
Cor(X,Y) = E[(X-ux)(Y-uy)] / 1.5

$$= \frac{[(1-2)(0.5-2) + (2-2)(2-2) + (3-2)(3.5-2)]/3}{1.5}$$

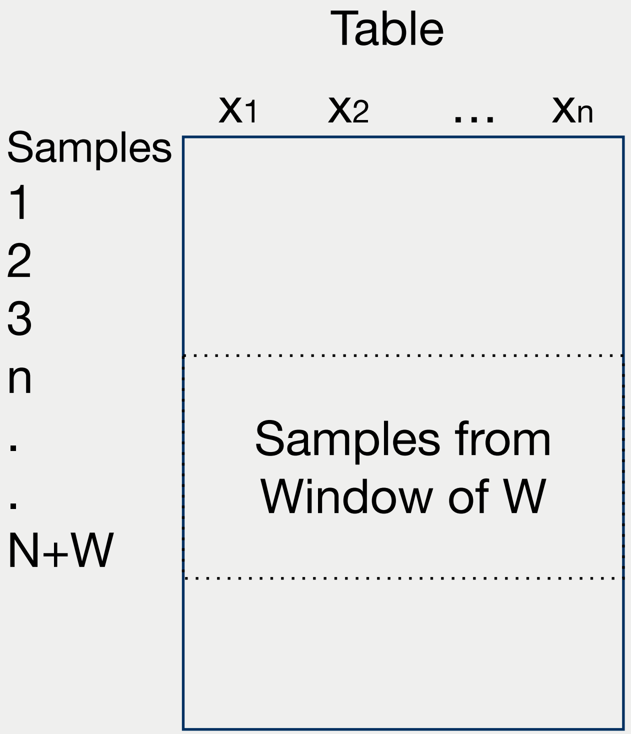
= 1.5 + 0 + 1*1.5 / (3* 1.5) = 1/1.5 = .67

Berkeley SCET

Correlation Matrix



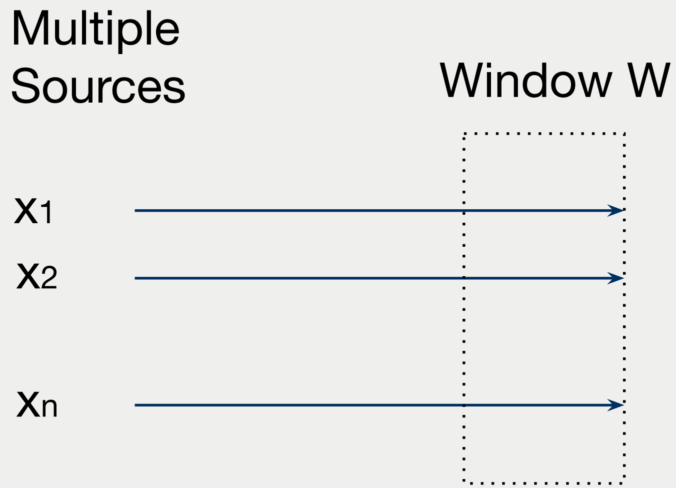
Many samples, or a sequence in time $x_1(t)$, $x_2(t)$, .. $x_n(t)$



To estimate from data:

a) Use all samples ever collected
b) Use window size of W samples of each to estimate a recent Corr Matrix

Correlation Matrix



$$\begin{pmatrix} \mathrm{Corr}(x1,x1) \; \ldots\ldots\ldots & \mathrm{Corr}(x1,xn) \\ & \\ \mathrm{Corr}(xn,x1) & \mathrm{Corr}(xn,xn) \end{pmatrix}$$

Corr(x1,x1) = 1

Corr(x2, x1) = a number from -1 to 1

To estimate from data:

a) Use all samples ever collected
b) Use window size of W samples of each to estimate recent Corr Matrix

Code Example:

Correlations of Rows with Numpy

```
Import numpy as np

# ignore line formatting
x = np.array(
        [[0.1, .32, .2,  0.4, 0.8],
        [.23, .18, .56, .61, .12],
        [.9,   .3,  .6,  .5,  .3],
        [.34, .75, .91, .19, .21]])

np.corrcoef(x)
Out[4]: array([
[ 1.        , -0.35153114, -0.74736506, -0.48917666],
[-0.35153114,  1.        ,  0.23810227,  0.15958285],
[-0.74736506,  0.23810227,  1.        , -0.03960706],
[-0.48917666,  0.15958285, -0.03960706,  1.        ]
])
```

- Here each row is a vector of length 5
- There are 4 vectors
- Correlation matrix is 4 x 4

- If you want the correlation of the columns, just use transpose

- np.corrcoef (np.transpose(x))

- For a window, use a slice:
- window = x[0:4,3:5] for the last
- two columns

Berkeley SCET

Correlation of Features
from Different Sources

	mpg	disp	hp	drat	wt	qsec
Mazda RX4	21.0	160	110	3.90	2.620	16.46
Mazda RX4 Wag	21.0	160	110	3.90	2.875	17.02
Datsun 710	22.8	108	93	3.85	2.320	18.61
Hornet 4 Drive	21.4	258	110	3.08	3.215	19.44
Hornet Sportabout	18.7	360	175	3.15	3.440	17.02
Valiant	18.1	225	105	2.76	3.460	20.22

pandas.DataFrame.corr

`DataFrame.corr(method='pearson', min_periods=1)` [source]

Compute pairwise correlation of columns, excluding NA/null values

Parameters: **method** : *{'pearson', 'kendall', 'spearman'}*

- pearson : standard correlation coefficient
- kendall : Kendall Tau correlation coefficient
- spearman : Spearman rank correlation

min_periods : *int, optional*

Minimum number of observations required per pair of columns to have a valid result. Currently only available for pearson and spearman correlation

Returns: **y** : *DataFrame*

Pandas Table
Use corr() method

dataframe.corr()

	mpg	disp	hp	drat	wt	qsec
mpg	1.00	-0.85	-0.78	0.68	-0.87	0.42
disp	-0.85	1.00	0.79	-0.71	0.89	-0.43
hp	-0.78	0.79	1.00	-0.45	0.66	-0.71
drat	0.68	-0.71	-0.45	1.00	-0.71	0.09
wt	-0.87	0.89	0.66	-0.71	1.00	-0.17
qsec	0.42	-0.43	-0.71	0.09	-0.17	1.00

Correlation Types: Pearson, Kendal, Spearman

pandas.DataFrame.corr

`DataFrame.corr(method='pearson', min_periods=1)`

Compute pairwise correlation of columns, excluding NA/null values

Parameters: **method** : *{'pearson', 'kendall', 'spearman'}*

- pearson : standard correlation coefficient
- kendall : Kendall Tau correlation coefficient
- spearman : Spearman rank correlation

min_periods : *int, optional*

Minimum number of observations required per pa

Currently only available for pearson and spearma

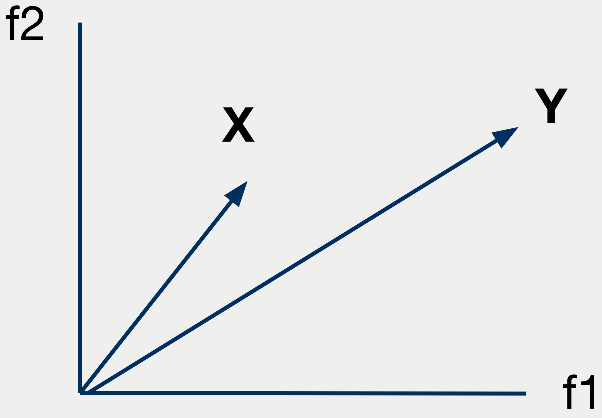
Returns: **y** : *DataFrame*

Data Table

X Y

--

5	7
8	10
14	7
15	12
…	…
…	…
…	…
…	…
…	…



Pearson: Understanding Correlation in a different way

Use n dimensions

X • Y = |X| |Y| Cos Θ

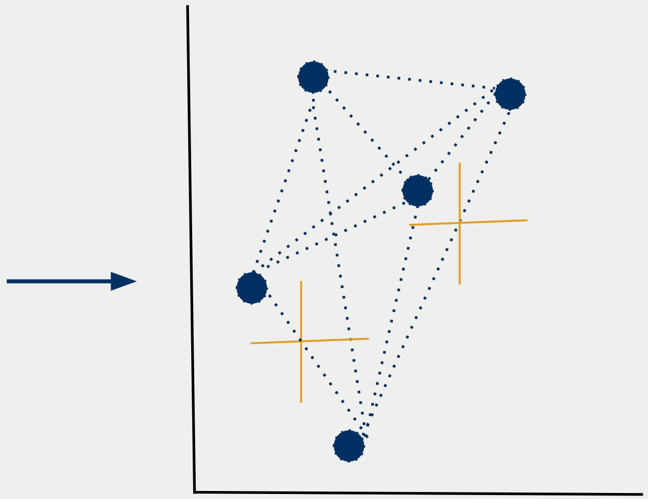
Pandas will create a correlation matrix with “columns”

```
In [15]: frame = pd.DataFrame(np.random.randn(1000, 5), columns=['a', 'b', 'c', 'd', 'e

In [16]: frame.ix[::2] = np.nan

# Series with Series
In [17]: frame['a'].corr(frame['b'])
Out[17]: 0.013479040400098775

In [18]: frame['a'].corr(frame['b'], method='spearman')
Out[18]: -0.0072898851595406371

# Pairwise correlation of DataFrame columns
In [19]: frame.corr()
Out[19]:
          a         b         c         d         e
a  1.000000  0.013479 -0.049269 -0.042239 -0.028525
b  0.013479  1.000000 -0.020433 -0.011139  0.005654
c -0.049269 -0.020433  1.000000  0.018587 -0.054269
d -0.042239 -0.011139  0.018587  1.000000 -0.017060
e -0.028525  0.005654 -0.054269 -0.017060  1.000000
```

Kendall Correlation

List of (x,y) points

No	X	Y
1	2	3
2	4	6
3	3	8
4	9	12



N points

N(N -1) / 2 pairs of x,y points

- Concordant pairs: for (x_i, y_i) and (x_j, y_j), where $i \neq j$,

  $x_i > x_j$ and $y_i > y_j$ or $x_i < x_j$ and $y_i < y_j$

- Disconcordant pairs: when the above is not true

  if $x_i > x_j$ and $y_i < y_j$

  or if $x_i < x_j$ and $y_i > y_j$

The Kendall τ coefficient is defined as:

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{n(n-1)/2}$$

Berkeley SCET

Spearman Correlation

Data (x=IQ,y=TV)

IQ, X_i	Hours of TV per week, Y_i
106	7
86	0
100	27
101	50
99	28
103	29
97	20
113	12
112	6
110	17

$$r_s = \rho_{\mathrm{rg}_X,\mathrm{rg}_Y} = \frac{\mathrm{cov}(\mathrm{rg}_X, \mathrm{rg}_Y)}{\sigma_{\mathrm{rg}_X}\sigma_{\mathrm{rg}_Y}}$$

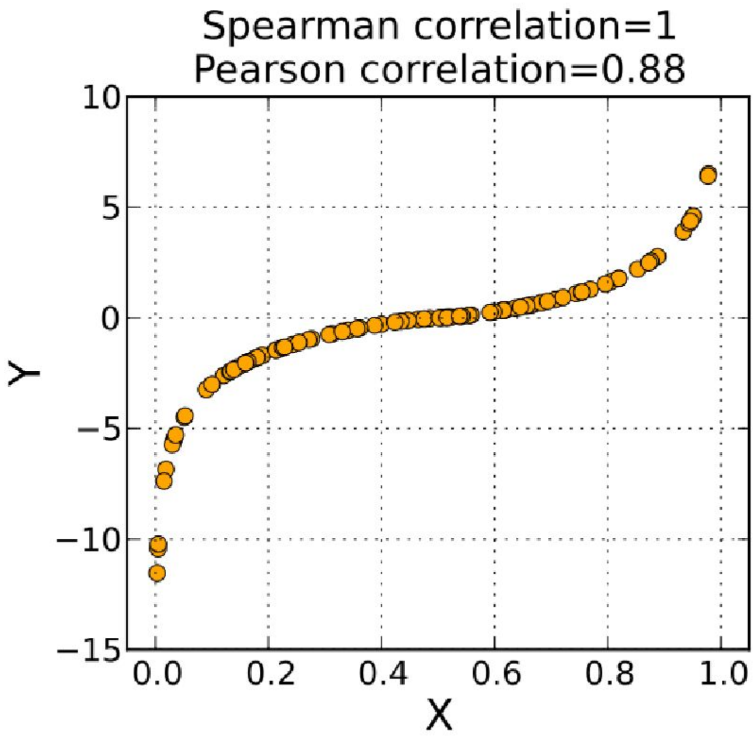
where

- ρ denotes the usual Pearson correlation coefficient, but applied to the rank variables.
- $\mathrm{cov}(\mathrm{rg}_X, \mathrm{rg}_Y)$ is the covariance of the rank variables.
- σ_{rg_X} and σ_{rg_Y} are the standard deviations of the rank variables.

x	y	rgx	rgy		
97	20	2	6	−4	16
99	28	3	8	−5	25
100	27	4	7	−3	9
101	50	5	10	−5	25
103	29	6	9	−3	9
106	7	7	3	4	16
110	17	8	5	3	9
112	6	9	2	7	49
113	12	10	4	6	36

Order rows by X and Index X and Y in increasing order

Then find Pearson Correlation of (rgx,rgy)



A Spearman correlation of 1 results when the two variables being compared are monotonically related, even if their relationship is not linear. This means that all data-points with greater x-values than that of a given data-point will have greater y-values as well. In contrast, this does not give a perfect Pearson correlation.

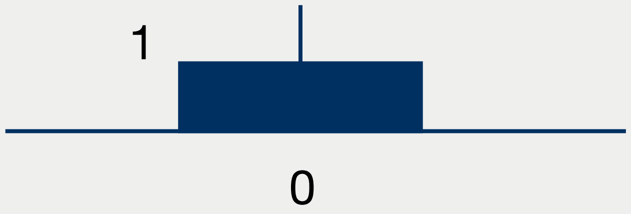
Berkeley SCET

Correlation with Time Series Data Sources

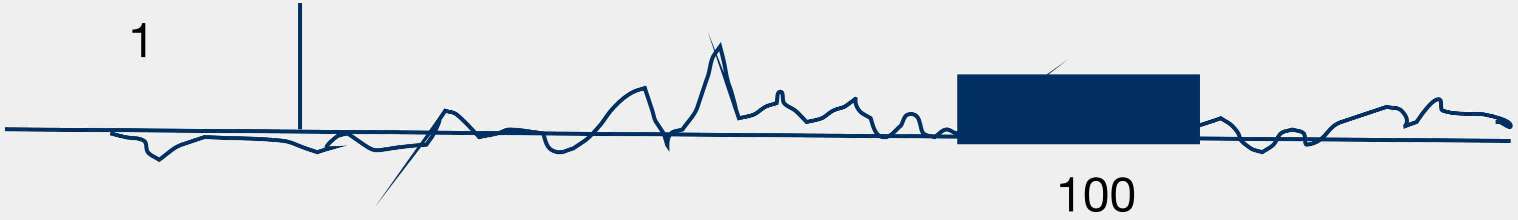
Berkeley SCET

Correlation Matrix with multiple sources and time segments

Suppose this is x1
as an array of numbers 0 0 1 1..1 0 0 0



Suppose this is x2

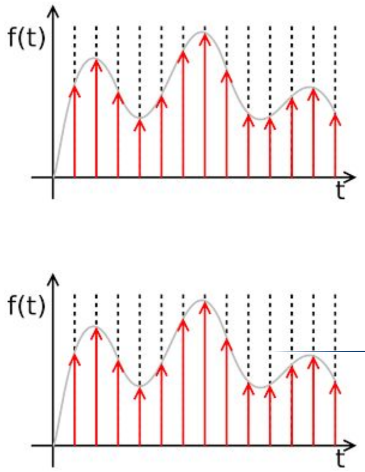


What is np.corr(x1,x2[n:n+w])?

Berkeley SCET

Approaches to the Data Sequences from Multiple Sources in Tables

A



D

Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …

One row for each source
A, B, C, D..

Eg Numpy arrays

Time ->

A:	x1, x2, x3, …
B:	x1, x2, x3, …
C:	x1, x2, x3, …
D:	x1, x2, x3, …

One column for each series:
A , B, C, D..

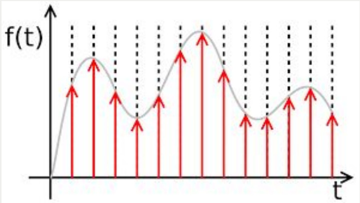
eg Pandas
(add rows with every new time sample)

	A	B	C
1	x1	x1	x1
2	x2	x2	x2
3	x3	x3	x3
.	.	.	.
.	.	.	.

Berkeley SCET

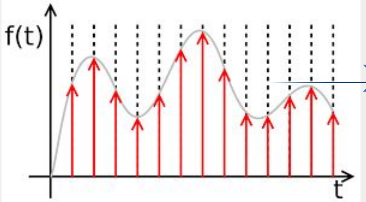
Approaches to the Data Sequences in Tables

A



.

.



D

Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …

One row for each source
A, B, C, D..

Eg Numpy arrays

Time ->

A: x1, x2, x3, …
B: x1, x2, x3, …
C: x1, x2, x3, …
D: x1, x2, x3, …

One column for each series:
A , B, C, D..

eg Pandas
(add rows with every new time sample)

	A	B	C
1	x1	x1	x1
2	x2	x2	x2
3	x3	x3	x3
.	.	.	.
.	.	.	.

Preprocess:
Filter/
Convolution
Correlation
Mean, var,
stats

Windows of time

Features

If ML:
Table of (X,Y)
for
Prediction
Classification

Data Input and Storage

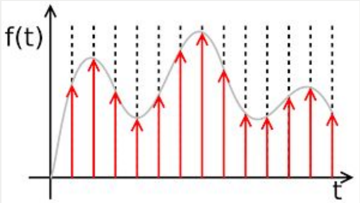
Preprocessing

ML for Decisions

Berkeley SCET

Approaches to the Data Sequences in Tables

A



.
.

D

Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …

One row for each source A, B, C, D..

Eg Numpy arrays

Time ->

A: x1, x2, x3, …
B: x1, x2, x3, …
C: x1, x2, x3, …
D: x1, x2, x3, …

One column for each series: A , B, C, D..

eg Pandas (add rows with every new time sample)

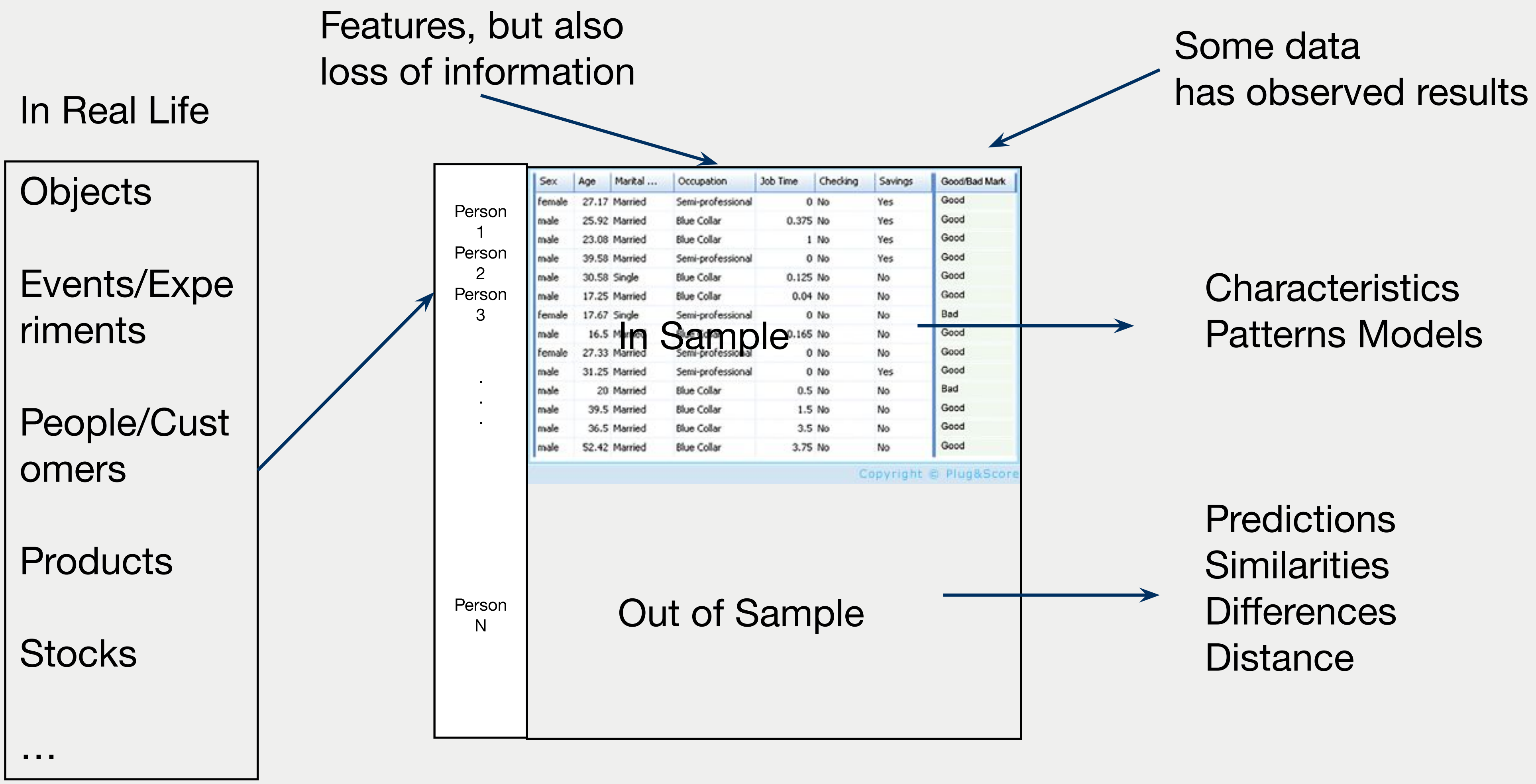
	A	B	C
1	x1	x1	x1
2	x2	x2	x2
3	x3	x3	x3
.	.	.	.
.	.	.	.

Data Input and Storage

						Labeled
	Cor(w/A)	Cor(w/B)	St.Dev.	Cor(w/A[n-20])		Condition
A	#	#	#	#		Danger
B	#	#	#	#		Safe
C	#	#	#	#		Safe
D	#	#	#	#		Warning
# = number obtained from preprocessing						

Example: pre-processed statistics can be used for in ML predictions

A High-Level Framework



Sex	Age	Marital ...	Occupation	Job Time	Checking	Savings	Good/Bad Mark
female	27.17	Married	Semi-professional	0	No	Yes	Good
male	25.92	Married	Blue Collar	0.375	No	Yes	Good
male	23.08	Married	Blue Collar	1	No	Yes	Good
male	39.58	Married	Semi-professional	0	No	Yes	Good
male	30.58	Single	Blue Collar	0.125	No	No	Good
male	17.25	Married	Blue Collar	0.04	No	No	Good
female	17.67	Single	Semi-professional	0	No	No	Bad
male	16.5	Married	Blue Collar	0.165	No	No	Good
female	27.33	Married	Semi-professional	0	No	No	Good
male	31.25	Married	Semi-professional	0	No	Yes	Good
male	20	Married	Blue Collar	0.5	No	No	Bad
male	39.5	Married	Blue Collar	1.5	No	No	Good
male	36.5	Married	Blue Collar	3.5	No	No	Good
male	52.42	Married	Blue Collar	3.75	No	No	Good

END OF SECTION